Mathematics Applications Year 12
Test 4   2019
Calculators allowed, 1 page of notes

SHENTON COLLEGE

NAME: _____ Solutions _____

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Time: 50 min   Marks: _____ /49   ☐ Units
Show all working in the spaces provided.   ☐ Rounding
Full marks may not be awarded without sufficient working.

Question 1   [6: 3, 3 marks]

Digby, the gardener at Kings Park, wishes to renew the network of reticulation pipes which water the garden beds located at various buildings around the Park. The table below shows the existing network of pipes, in metres.

(a)   On the table below show working, using Prim's Algorithm, to find the minimum length of piping to replace to connect the garden beds. State this distance.

1250m ✓ (FT) use

	Botanic Gardens	Frasers Restaurant	Aspects Gift Shop	Gazebo	Digby's Shed	Sticky Beaks Cafe	Zamia Cafe
Botanic Gardens	--	120	60	100	--	425	--
Frasers Restaurant	120	--	35	60	--	--	--
Aspects Gift Shop	60	35	--	45	60	--	--
Gazebo	100	60	45	--	110	400	800
Digby's Shed	--	--	60	110	--	585	750
Sticky Beaks Cafe	425	--	--	400	585	--	650
Zamia Cafe	--	--	--	800	750	650	--

−1 errors.   ✓circles   ✓lines   ✓arrows   −1 errors

(b)   Use your working from the table to draw a network which shows the minimum connections.
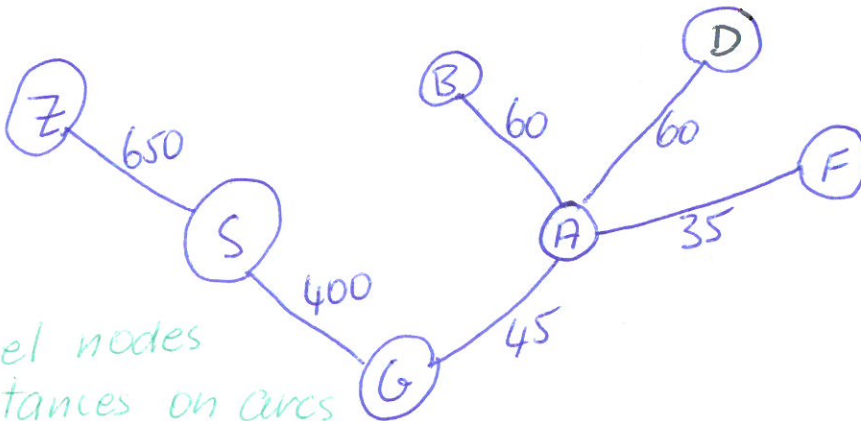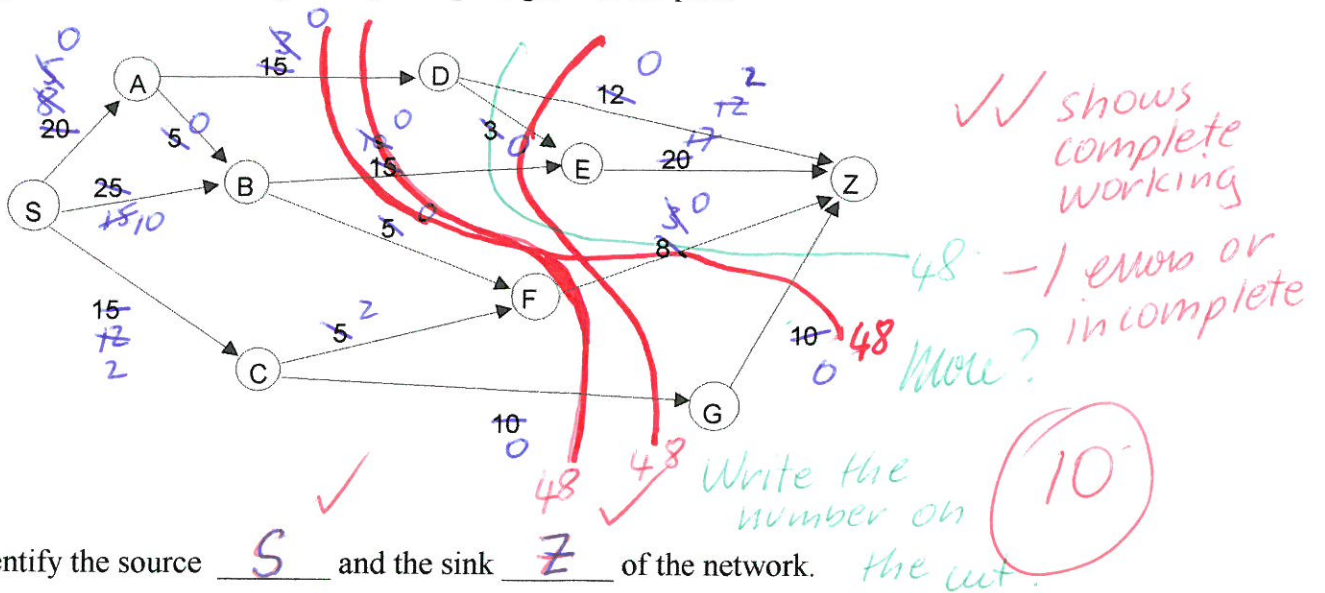

use FT

✓ tree
✓ nodes named
✓ arcs numbered

−1 errors

− label nodes
− distances on arcs

Question 2 [10: 2, 1, 4, 1, 2 marks]

Each year during Spring, grain is harvested across Western Australia where it is stored in Silos before making its way to Port for overseas distribution. The network below shows the number of trucks per day, which may travel on the country roads, taking the grain to the port.

(a) Identify the source __S__ and the sink __Z__ of the network.

shows complete working
48 -1 errors or incomplete.
More?
Write the number on the cut (10)

(b) Explain why the maximum flow cannot exceed 60 trucks per day.

There are only 60 trucks leaving the source each day. ✓ *allow (50) into sink and source.* *60*

(c) By listing your paths determine the maximum number of trucks possible for each day. Clearly show your working on the diagram.

$$
\begin{array}{ll}
SADZ & 12 \\
SADEZ & 3 \\
SA\ BEZ & 5 \\
SBEZ & 10 \\
SBFZ & 5 \\
SCFZ & 3 \\
SCGZ & 10 \\
\hline
& 48
\end{array}
$$
✓

∴ 48 trucks / day
✓ *for 48 units dealt with once!*

(d) Show clearly on the above network, a minimum cut which matches your calculated maximum flow.

More than one solution. ✓

(e) If you could upgrade one road to increase the number of trucks per day, which road would you choose? Clearly state the new number of trucks per day limit.

BE
OR FZ ✓ *correct road*
either would change the max trucks to (50) per day. ✓ *correct max trucks* *(up by 2)*

Question 3   [8:1, 3, 1, 1, 2 marks]

Josh Kennedy, from the West Coast Eagles, wishes to retire at the end of the 2019 season. He has savings of $500 000 from his successful football career and wishes to deposit this amount into an annuity which which will earn interest of 2.9% p.a. compounded annually. He plans to study for a number of years and to do this he will withdraw $50 000 at the end of the first year, $51 000 at the end of the second year, $52 020 at the end of the third year, and so on, increasing each amount by 2%.

(a)   What will be his account balance after one year?

$$500\,000 \times 1.029 - 50\,000 = \$464\,500 \checkmark$$

⑧

(b)   Write a recurrence relation which Josh can use to model his annuity.

$$T_{n+1} = 1.029\,T_n - 50\,000 \times 1.02^n \qquad T_0 = 500\,000$$

-1 it not a recurrence relation

correct interest.

correct indexing

correct T_0.

(c)   What will be the balance of his annuity immediately after the eighth withdrawal?

$$T_8 = \$154\,565.13 \quad \checkmark \text{ correct } T \text{ value.}$$

without indexing
$T_8 = \$185\,540.06$

(d)   How long will it take for the balance of the account to reach zero?

reads ✓ from table

11 years.

$$T_{10} = 43\,623.39$$
$$T_{11} = -16\,061.25$$

(e)   How much will the last payment be?

meant to be ✓ ✓ Accounts for final increase in interest   too much

①

$$50\,000 \times 1.02^{10} = 60\,949.72 - 16\,061.25$$

$$= \$44\,888.47$$

✓ Calculates final payment!

②

OR $43\,623.39 \times 1.029$

$$= \$44\,888.47$$

Uses T_{10} ✓

$\times 1.029$ ✓

Question 4 [5: 3, 2 marks]

Water pipes from Mundaring Weir, are connected to small towns throughout the Wheatbelt, and are represented as a network (in kilometres) given below.

Not so well done!

(a) Show, on the diagram, the minimum amount of pipe which has to be laid for water to be connected to every town? State this amount.
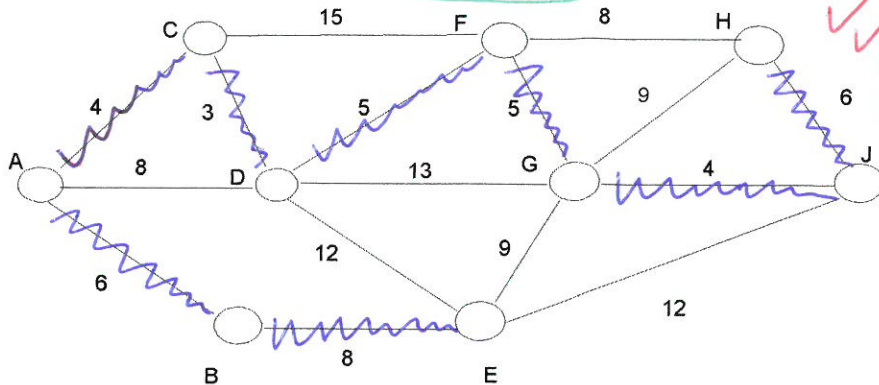
(−1 each error)

✓ diagram correct

```
8
6
4
3
5
5
4
6
___
41 km
```

✓ correct total.

(b) Town B has a large dam and the water company has decided it is uneconomical to provide water pipes to that town. How many kilometres of piping is the company saving if Town B is now excluded?

$$41 - 14 + 9 = 36 \text{ km}$$

✓ new total.

Saves 5km

✓ amt. saved.

Question 5 [4 marks] The following activities are involved in the completion of a project. In the space below, construct a project network for these activities.

9⃝

Activity	Time needed (in hours)	Immediate Predecessor(s)
A	20	-
B	12	A
C	10	-
D	7	C
E	11	C
F	5	B,E
G	10	B,E
H	8	G
I	4	F
J	5	D
K	9	J,H

(−1 each error)

✓ numbers
✓ letters
✓✓ network + arrows

Must start the network →

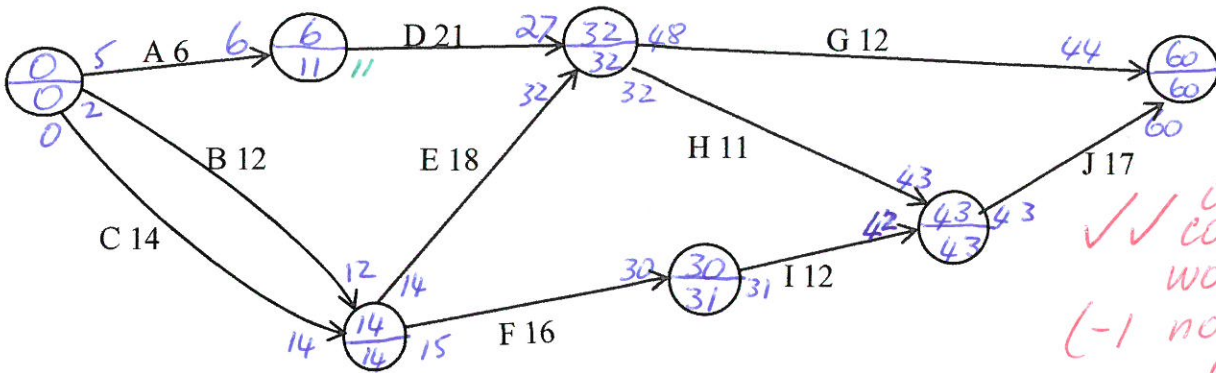A12   B12   F5   I4
C10   E11   G10   H8   K9
D7   J5

FINISH THE NETWORK!
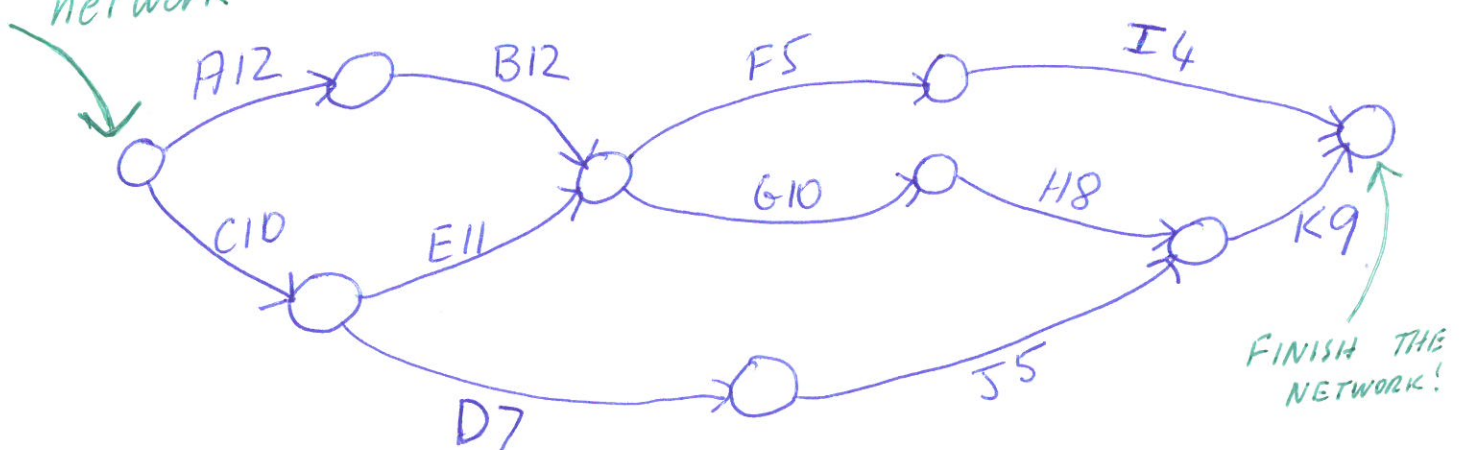
Question 6   [11 : 1, 4, 1, 2, 1, 2 marks]

The following project network shows completion time of activities **A** to **J** in minutes.

[Handwritten annotations in red:] ✓✓ correct. complete working (−1 no numbers on arcs)

(a)   How many activities are required to be completed for this project?

10 ✓ correct number.

(b)   State the critical path and the minimum completion time for the project, after showing full working on the network above.

Critical Path is C E H J ✓
FT from Diag.

60 minutes ✓ from diag.

(c)   Determine the float time of activity D?

5 minutes ✓ *(11)*

(d)   If activity J could be completed in 10 *minutes* how would the critical path and minimum completion time be affected?

CP not affected, ✓ correct statement.

CT is now 53 mins (7mins less) ✓ accept either.

The project commenced at 8.30am on Monday.

(e)   What would be the latest starting time for activity D?

8·30 + 11 mins ⇒ 8·41 am ✓ adds 15 minutes

(f)   If only activity D was delayed by 8 minutes, what time would the project be completed?

+ 3 mins, so 63 mins
correct amt delayed ∴ 9:33am Monday.
✓ (9.33am enough)

Question 7   [3 marks]

The Country Women's Association wants to set up an honorary scholarship for a high performing young rural student each year, to help pay for their living costs while at University. How much will they need to put into an account (rounded to the nearest $1 000) to ensure that they can draw $12 500 from an account perpetually each year if the account has an interest rate of 4.5% p.a compounding monthly?

$$i_e = \left(1 + \frac{0.045}{12}\right)^{12} - 1 = 0.04593983 \text{ or } 4.594\%/pa. \text{ effective.}$$

✓ correct i_e.

OR

N   12
EFF   4.59398
APR   4.5

OR

$$\left(P + \$12500 = P\left(1 + \frac{0.045}{12}\right)^{12}\right) \underline{SOLVE}.$$

$272095.07

$$x \times 0.04594 = 12\,500$$

$$x = \frac{12\,500}{0.04595}$$

$$= \$272\,094.04$$

✓ applies correct formula

∴ needs $273\,000 (must round up)

✓ rounds up.

Question 8   [2 marks]

A youthful 60 year old teacher wants to retire and be able to afford to live until she is at least 98 years old, so that she can go trekking and enjoy her family. She will need to withdraw $65 000 once a year. What is the minimum amount she would need to have saved up by the time she retires if her bank gives her an interest rate of 4.9% p.a. compounded quarterly?

N 38
I 4.9
PV [-1097 757.53]
PMT 65 000
FV 0
P/Y 1
C/Y 4

✓ compounds quarterly
   1 payment

needs to have saved

$1 097 757.53

✓ correct answer from FINANCIAL

⑤